CLASS – 11 (MP BOARD)
SUB. : MATHEMATICS
CHAPTER – 04
MEASUREMENT OF ANGLES

Que. 1 : Express the following angles in radian measure :

(a) 45^0 (b) 520^0 (c) 110^0 (d) -310^0 (e) 630^0
(f) $-22^0 30'$ (g) $5^0 14' 36''$ (h) $15^0 22' 30''$

Solution :

(a) $45^0 = (45 \times \frac{\pi}{180})^c = (\frac{\pi}{4})^c$

(b) $520^0 = (520 \times \pi/180)^c = (26\pi/9)^c$

(c) $110^0 = (110 \times \pi/180)^c = ((11\pi/18)^c$

(d) $-310^0 = (-310 \times \pi/180)^c = -(31\pi/18)^c$

(e) $630^0 = (630 \times \pi/180)^c = (7\pi/2)^c$

(f) $-22^0 30' = -(22\frac{1}{2})^0 = -(45/2)^0 = -(45\pi/360)^c = -(\pi/8)^c$

(g) $36'' = (36/60)' = (3/5)'$

$15' 36'' = (15\frac{3}{5})' = (78/5)' = (78/300)^0 = (13/50)^0$

$5^0 15' 36'' = (5\frac{13}{50})^0 = (263/50)^0 = (\frac{263}{50} \times \frac{\pi}{180})^c = (263\pi/9000)^c$

(h) $30'' = (30/60)' = (1/2)'$

$22' 30'' = (22\frac{1}{2})' = (45/2)' \ (45/120)^0 = (3/8)^0$

$15^0 22' 30'' = (15\frac{3}{8})^0 = (123/8)^0 = (\frac{123}{8} \times \frac{\pi}{180})^c = (123\pi/1440)^c$

Que. 2 : Convert the following radian into degree measure :

(a) $(9\pi/5)^c$ (b) $(-5\pi/6)^c$ (c) -2^c
(d) 1^c (e) $(1/4)^c$

Solution : (a) $(9\pi/5)^c = (\frac{9\pi}{5} \times \frac{180}{\pi})^0 = 324^0$

(b) $(-5\pi/6)^c = -(\frac{5\pi}{6} \times \frac{180}{\pi})^0 = -150^0$

(c) $-2^c = (-2 \times \frac{180}{\pi})^0 = (\frac{-2\times180\times7}{22})^0 = -(\frac{2520}{22})^0 = -114^0 32' 44''$

(d) $1^c = (180/\pi)^0 = (\frac{180\times7}{22})^0 = (\frac{1260}{22})^0 = 57^0 16' 22''$

(e) $(1/4)^c = (\frac{1}{4} \times \frac{180}{\pi})^0 = (\frac{1}{4} \times \frac{180 \times7}{22})^0 = (\frac{1260}{88})^0 = 14^0 19' 5''$

Que. 3 : What must be the radius of a wheel if an arc 24π cm long subtends an angle of 72^0 at the centre ?

Solution : Given : Arc = 24π cm, Angle = $72^0 = 72 \times (\pi/180) = (2\pi/5)^c$
Let the radius of the wheel be 'r' cm., we know that
Angle = Arc / Radius
⇨ 2π/5 = 24π / r
⇨ 2r = 120, therefore r = 60 cm.

Que. 4 : Find the length of an arc of a circle of radius 14 cm which subtends an angle of 36^0 at the centre ?

Solution : Given : Radius of circle = 14 cm,
Angle = $36^0 = 36\ (\pi/180) = (\pi/5)^c$
Let the length of the arc be 'l' cm
Since Arc = Angle × Radius
= (π/5) × 14 = 8.8 cm.
Therefore length of the arc = 8.8 cm.

Que. 5 : Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by arc of length 22 cm.

Solution : Given : Radius of circle = 100 cm
Length of arc = 22 cm
Angle = Arc / Radius = 22/100 = 11/50 Radians
Therefore the degree measure of the angle

$$= (\frac{11}{50} \times \frac{180}{\pi})^0 = (\frac{11}{50} \times \frac{180\times7}{22})^0 \ (12.6)^0 = 12^0 36'$$

Que. 6 : **Find the degree measure of the angle subtended at the centre of a circle of diameter 60 cm by an arc of length 16.5 cm.**

Solution : **Given :**
Diameter of circle = 60 cm
Radius of circle = 30 cm
Length of arc = 16.5 cm
Angle = arc / radius = 16.5 / 30 = 11/20 Radians
Therefore the degree measure of the angle

$$= \left(\frac{11}{20} \times \frac{180}{\pi}\right)^0 = (31.5)^0 = 31^0 30'$$

Que. 7 : **In a circle of diameter 28 cm, the length of a chord is 14 cm. Find the length of minor arc of the chord.**

Solution : **Given :**
Diameter of circle = 28 cm
Radius of circle = 14 cm
Length of chord = 14 cm
Since radius of circle and length of chord are equal, hence it forms an equilateral triangle i.e., angle subtended at the centre is 60^0 or $(\pi/3)^c$.
Length = angle × radius = $(\pi/3) \times 14 = 14\pi/3$ cm.

Que. 8 : **Find the length of the arc of a circle of radius 25 cm subtending a central angle of 15^0**

Solution : **Given :**
Radius of circle = 25 cm.
Angle = $15^0 = 15\pi/180 = \pi/12$ radians
Let the length of arc be 'l' cm.
Arc = angle × radius = $(\pi/12) \times 25$ = 275/42 = 6.55 cm.
Therefore length of arc = 6.55 cm.

Que. 9 : **If in two circles, area of same length subtend angles of 65^0 and 110^0 at the centre then find the ratio of their radii.**

Solution : **Suppose the length of arcs of both the circles be 'l' cm and their radii be r_1 and r_2 respectively.**
Given that : $\theta_1 = 65^0$ and $\theta_2 = 110^0$, then
$\theta_1 = 65^0 = (65\pi/180)^c$ and $\theta_2 = 110^0 = (110\pi/180)^c$
$\theta_1 / \theta_2 = (l/r_1)/(l/r_2)$ => $65/110 = r_2/r_1$ => $r_1/r_2 = 110/65 = 22/13$
Therefore the ratio of their radii = 22:13.

Que. 10 : Express the angular measurement of the angle of a regular :-
(i) Octagon (ii) Heptagon (iii) Duodecagon

Solution : **(i) OCTAGON :**

We know that the angle of a regular polygon having 'n' sides is equal to (2n-4)/n right angles.

Let θ_1 be the angle of octagon, then

θ_1 = (2x8 – 4)/8right angles=(12/8)right angles=(3/2)right angles

= (3/2) x 90^0 = 135^0 = (3π/4) radians.

(ii) HEPTAGON :

Let θ_2 be the angle of heptagon, then

θ_2 = (2x7 – 4)/7right angles=(10/7)right angles

= (10/7) x 90^0 = $(900/7)^0$ = (5π/7) radians.

(iii) DUODECAGON :

Let θ_3 be the angle of duodecagon, then

θ_3 = (2x12 – 4)/12right angles=(5/3)right angles

= (5/3) x 90^0 = $(150)^0$ = (5π/6) radians.

Que. 11 : A wheel makes 540 revolutions in one minute. How many radians does it turn in one second ?

Solution : Number of revolutions by wheel in 1 minute = 540

Therefore number of revolutions by wheel in 1 second

= 540/60 = 9

Angle moved in 1 revolution = 360^0 = $2\pi^c$

Therefore angle moved in 9 revolutions = 9 x $2\pi^c$ = $18\pi^c$

Que. 12 : In a right angled triangle, the difference between two acute angles is $(\pi/15)^c$. Find the angles in degrees and radians.

Solution : Let 'x' and 'y' be the two acute angles and x>y.

Since the triangle is right angled, hence the sum of two acute angles = 90^0 i.e., x + y = 90^0 ---------------- (1)

And x – y = $(\pi/15)^c$ (given)

Or x – y = 12^0 ---------------- (2)

From equations (1) and (2), we get

x = 51^0 and y = 39^0

Que. 13 : The difference of the measure of acute angles of right angle triangle is

(2/5)π radians. Find the acute angles in degrees and radians.

Solution : Let 'x' and 'y' be two acute angle and x>y

Since in a right angled triangle, the sum of acute angles is 900 or (π/2)c

So, $x + y = \pi/2$ ---------------------------(1)

Also given that $x - y = 2\pi/5$ ---------------- (2)

On adding equation (1) and (2), we get

$2x = (\pi/2) + (2\pi/5) = 9\pi/10$ or $x = 9\pi/20$ radians

Putting value of 'x' in equation (1), we get

$y = \pi/20$ radians

Now, the degree measure of the angles 'x' and 'y' are

$x = (9\pi/20 \times 180/\pi)^0 = 81^0$ and

$y = (\pi/20 \times 180/\pi)^0 = 9^0$.

Que. 14 : The angles of a triangle are in A.P. and the greatest angle is 84^0, find all the angles in radians.

Solution : Let the three angles be $\alpha - \beta, \alpha, \alpha + \beta$

We know that the sum of all angles of a triangle is 180^0

Therefore, $\alpha - \beta + \alpha + \alpha + \beta = 180^0$

$3\alpha = 180^0 \Rightarrow \alpha = 60^0$

It is given in the question that greatest angle is 840

Then $\alpha + \beta = 84^0 \Rightarrow \beta = 84^0 - 60^0 = 24^0$

Third angle $\alpha - \beta = 60^0 - 24^0 = 36^0$.

Now, the radian measure of all the angles are

Radian measure of $60^0 = 60 \times \pi/180 = \pi/3$ radians

Radian measure of $84^0 = 84 \times \pi/180 = 7\pi/15$ radians

Radian measure of $36^0 = 36 \times \pi/180 = \pi/5$ radians.

Que. 15 : Find the angle through which a pendulum swings if its length is 75 cm and the bob describe an arc of

(a) 10 cm (b) 15 cm

Solution :

(a) Given : radius = 75 cm and length = 10 cm

Let θ_1 be the angle made by the swing of bob

We know that

Angle = arc / radius $\Rightarrow \theta_1 = 10/75 = 2/15$ radians

(b) Given : radius = 75 cm and length = 15 cm

Let θ_2 be the angle made by the swing of bob

We know that

Angle = arc / radius => θ_2 = 15/75 = 1/5 radians

Que. 16 : The angles of a triangle are in A.P. and the greatest angle is 75^0. Find the least angle in radians.

Solution : Let the three angles of triangle be
A = α - β, B = α and C = α + β and let angle C be the greatest.
By angle sum property of a triangle, we know that
$A + B + C = 180^0$ i.e.,
$\alpha - \beta + \alpha + \alpha + \beta = 180^0$
$3\alpha = 180^0 \Rightarrow \alpha = 60^0$
According to the given condition greatest angle is 75^0 i.e.,
$\alpha+\beta=75^0 \Rightarrow \beta=15^0$
Therefore the least angle is $\alpha-\beta = 60^0 - 15^0 = 45^0$

and the angle measure in radians = $45 \times \frac{\pi}{180} = \frac{\pi}{4}$ radians.

Que. 17 : The angles of a triangle are in A.P. and the number of degrees in the least is to the number of radians in the greatest is 60 : π. Find the angles in degrees.

Solution : Let the three angles of triangle be
A = α - β, B = α and C = α + β and angle A be the least and C be the greatest.
Radian measure of angle C = (α + β) × (π/180)
According to the given condition,

$$\frac{angle\ A\ in\ degrees}{angle\ C\ in\ radians} = \frac{60}{\pi}$$

$$\frac{\alpha-\beta}{\frac{(\alpha+\beta)\pi}{180}} = \frac{60}{\pi}$$

$$\frac{(\alpha-\beta)180}{(\alpha+\beta)\pi} = \frac{60}{\pi}$$

3(α-β) = (α+β)
3α - 3β = α + β
2α = 4β
α = 2β ----------------- (1)
since, $A + B + C = 180^0$
$\alpha - \beta + \alpha + \alpha + \beta = 180^0$
$3\alpha = 180^0$

$\alpha = 60^0$ --------------- (2)
from equation (1) and (2), we have
$\alpha = 60^0$ and $\beta = 30^0$
Therefore, angle A = $\alpha - \beta = 60^0 - 30^0 = 30^0$
angle B = $\alpha = 60^0$
angle C = $\alpha + \beta = 60^0 + 30^0 = 90^0$
Therefore the degree measures of the angles are 30^0, 60^0 and 90^0.

Que. 18 : A satellite is revolving round the earth in a circular orbit at a distance of a 10^4 km. What distance will it cover in revolving through 60^0 ?

Solution : Here, angle $\theta = 60^0 = (\pi/3)$ radians
And radius r = 10^4 kms.
Let the satellite travels a distance 'l'
Then, angle (θ) = $\frac{\text{length of arc } l}{\text{radius } r}$

Or length of arc (l) = angle (θ) × radius (r)
$l = (\pi/3) \times 10^4 = \frac{22 \times 10000}{7 \times 3} = 10476.19$ kms.

Que. 19 : Find the angle between the minute and hour hand of the clock when the time is 10:30.

Solution : Angle traced by the hour hand in 12 hours = 360^0
Therefore the angle traced by hour hand in 10 hrs. 30 minutes
- **⇨ 10 + (30/60) hrs.**
- **⇨ 10 + (1/2) hrs = 21/2 hrs**
- **⇨ $(360/12 \times 21/2)^0 = 315^0$**

Also, angle traced by minute hand in 1 hour (60 minutes) = 360^0
Therefore angle traced by minute hand in 30 minutes
$= (360 \times 30 / 60)^0 = 180^0$
Therefore required angle between two hands = $315^0 - 180^0 = 135^0$.

Que. 20 : If the angle between the two hands of the clock is 100^0 and the time is between 7 and 8, then find the correct time.

Solution : When the clock reads 7 o'clock, the angle between two hands
$= 7 \times 30^0 = 210^0$
If the required time is 7 hrs. 'x' minutes, then
$210^0 + (x/60) \times 30^0 - 6x^0 = \pm 100^0$
As the hour hand revolves 30^0 in one hour and minute hand moves 360^0 in one hour (i.e., 6^0 in 1minute)

$210 + (x/2) - 6x = \pm 100$

- ⇨ **$(x/2) - 6x = -210 \pm 100$**
- ⇨ **$-(11/2)\, x = -110$ or -310**
- ⇨ **$x = 20$ or $(620/11)$**

But (620/11) is not a whole number and clock cannot read this time. Therefore the time shown by the clock is 7:20.