Saket Shishu Ranjan Higher Secondary School Vidisha

Class 11: Maths
Chapter – 2: Relations And Functions

1. **Cartesian Product of Sets**
2. **Relations**
3. **Functions**

- **Ordered pair** A pair of elements grouped together in a particular order. Clearly, $(a, b) \neq (b, a)$.

- **Cartesian product of two sets** A and B is given by A × B = {(a, b): a ∈ A, b ∈ B}.

In particular **R × R** = {(x, y): x, y ∈ R} and **R × R × R** = (x, y, z): x, y, z ∈ R}

- If (a, b) = (x, y), then a = x and b = y.
- If n(A) = p and n(B) = q, then n(A × B) = pq.
- A × φ = φ
- In general, A × B ≠ B × A.

- **Relation**: Relation A relation R from a set A to a set B is a subset of the Cartesian product A × B obtained by describing a relationship between the first element x and the second element y of the ordered pairs in A × B, i.e., $\mathrm{R} \subseteq \mathrm{A} \times \mathrm{B}$.

- **Number of Relations**: Let A and B be two non-empty finite sets, containing m and n elements respectively, then the total number of relations from A to B is 2^{mn}

- **Domain**: The domain of R is the set of all first elements of the ordered pairs in a relation R. Domain R = $\{a : (a, b) \in \mathrm{R}\}$.

- The image of an element x under a relation R is given by y, where (x, y) ∈ R,

- **Range**: The range of the relation R is the set of all second elements of the ordered pairs in a relation R. Range R = $\{b : (a, b) \in \mathrm{R}\}$.

- **Function**: Function A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B. We write f: A→B, where f(x) = y.

- **Domain and Co-domain**: The set A is called the domain of function f and the set B is called the co-domain of f.

- **Range**: If f is a function from A to B, then each element of A corresponds to ine and only one element of B, whereas every element in B need not be the image of some x in A. The subset of B containing the image of elements of A is called the range of the function. The range of f is denoted by $f(\mathrm{A})$. Mathematically, we write: $f(\mathrm{A}) = \{f(x) : x \in \mathrm{A}\}$

- **Image**: If the element x of A corresponds to $y \in B$ under the function f, then we say that y is the image of x under f and we write, $f(x) = y$.
- **Pre-image**: If $f(x) = y$, then x is pre-image of y.

PRACTICE QUESTIONS 1(A)

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

Ans. Here $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

$\Rightarrow \frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$ and $y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3}$ and $y=\frac{3}{3}$

$\Rightarrow x=2$ and $y=1$

2. If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B).

Ans. Number of elements in set A = 3 and Number of elements in set B = 3

$\therefore$ Number of elements in A × B = 3 × 3 = 9

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Ans. Given: G = {7, 8} and H = {5, 4, 2}

$\therefore$ G×H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

And H × G = {(5, 7), (4, 7), (2, 7), (5, 8), (4, 8), (2, 8)}

4. State whether the following statements are true or false.

(i) If P = $\{m, n\}$ and Q = $\{n, m\}$, then P × Q = $\{(m, n)(n, m)\}$.

(ii) If A and B are non-empty sets, then A $\times$ B is a non-empty set of ordered pairs (x, y) such that $x \in$ A and $y \in$ B.
(iii) If A = {1, 2}, B = {3, 4}, then $A \times (B \cap \phi) = \phi$
Ans. (i) Here P = $\{m, n\}$ and Q = $\{n, m\}$
Number of elements in set P = 2 and Number of elements in set Q = 2

$\therefore$ Number of elements in P $\times$ Q = 2 $\times$ 2 = 4
But P$\times$Q = $\{(m, n), (n, m)\}$ and here number of elements in P $\times$ Q = 2
Therefore, statement is false.

(ii) True
(iii) True

5. If A = $\{-1, 1\}$, find A $\times$ A $\times$ A.
Ans. Here A = $\{-1, 1\}$
A $\times$A = $\{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$
AxAxA= $\{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$
$\therefore$

6. If A $\times$ B = $\{(a, x), (a, y), (b, x), (b, y)\}$, find A and B.
Ans. Given: A $\times$ B = $\{(a, x), (a, y), (b, x), (b, y)\}$
$\therefore$ A = set of first elements = $\{a, b\}$ and B = set of second elements = $\{x, y\}$

7. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that:
(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(ii) A $\times$ C is a subset of B $\times$ D.
Ans. Given: A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}

(i) $B \cap C$ = {1, 2, 3, 4} $\cap$ {5, 6} = ϕ
$\therefore$ $A \times B \cap C = \{1, 2\} \times \phi = \phi$ ……….(i)

A $\times$ B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}
A $\times$ C = {(1, 5), (1, 6), (2, 5), (2, 6)
$\therefore$ (A$\times$B) $\cap$ (A $\times$ C) = ϕ ……….(ii)

Therefore, from eq. (i) and (ii), $A \times B \cap C$

= (A × B) ∩ (A × C)
(ii) A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

Therefore, it is clear that each element of A × C is present in B × D.
∴ A × C ⊂ B × D

8. Let A = {1, 2} and B = {3, 4}, write A × B. How many sub sets will A × B have?
Ans. Given: A = {1, 2} and B = {3, 4}
∴ A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}
Number of elements in A × B = 4
Therefore, Number of subsets of A×B = 2^4 = 16

9. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2)(z, 1)$ are in A × B, find A and B.
Ans. Here $(x, 1) \in A \times B$
$\Rightarrow x \in$ A and $1 \in$ B

$(y, 2) \in A \times B$

$\Rightarrow y \in$ A and $2 \in$ B

$(z, 1) \in A \times B$

$\Rightarrow z \in$ A and $1 \in$ B

But it is given that $n(A) = 3$ and $n(B) = 2$

∴ A = $\{x, y, z\}$ and B = {1, 2}

10. The Cartesian Product A × A has 9 elements among which are found $(-1, 0)$ and (0, 1). Find the set A and all elements of A × A.
Ans. Here $(-1, 0) \in A \times A$
$\Rightarrow -1 \in$ A and $0 \in$ A

$(0,1) \in A \times A$

$\Rightarrow 0 \in$ A and $1 \in$ A

$\therefore -1, 0, 1 \in$ A

But it is given that $n(A \times A) = 0$ which implies that $n(A) = 3$

$\therefore$ A = $\{-1, 0, 1\}$

AxA=
$\{(-1,-1),(-1,0),(-1,1),(0,-1),(0,0),(0,1),(1,-1),(1,0),(1,1)\}$

PRACTICE QUESTIONS 1(B)

1. Let A = {1, 2, 3, ……., 14}. Define a relation R from A to A by R = $\{(x,y): 3x - y = 0,$ where $x, y \in A\}$. Write down its domain co-domain and range.
Ans. Given: A = {1, 2, 3, ……….., 14}
The ordered pairs which satisfy $3x - y = 0$ are (1, 3), (2, 6), (3, 9) and (4, 12).

$\therefore$R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Domain = {1, 2, 3, 4}

Range = {3, 6, 9, 12}

Co-domain = {1, 2, 3, ……….., 14}

2. **Define a relation R on the set N of natural numbers R = $\{(x,y): y = x + 5, x$ is a natural number less than 4 and $x, y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.**
Ans. Given:
R= $\{(x, y: y = x + 5, x \text{ is a natural number less than } 4 : x, y \in N)\}$

Putting x = 1, 2, 3 in $y = x + 5,$ we get y = 6, 7, 8

$\therefore$R = {(1, 6), (2, 7), (3, 8)}

Domain = {1, 2, 3}

Range = {6, 7, 8}

3. A = {1, 2, 3 5} and B = {4, 6, 9}. Define a relation R from A to B by R = $\{(x, y):$ the difference between x and y is odd: $x \in A, y \in B\}$. Write R in roster form.

Ans. Given: A = {1, 2, 3, 5} and B = {4, 6, 9}, $x \in$ A, $y \in$ B

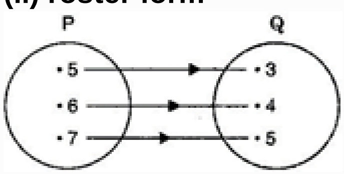
$\therefore x - y$ = (1 – 4), (1 – 6), (1 – 9), (2 – 4), (2 – 6), (2 – 9), (3 – 4), (3 – 6) (3 – 9),

(5 – 4), (5 – 6), (5 – 9)

$$\Rightarrow x - y = -3, -5, -8, -2, -4, -7, -1, -3, -6, 1, -1, -4$$

$\therefore$ R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6) (5, 4), (5, 6)}

4. Figure shows a relationship between the sets P and Q. Write this relation:
(i) in set-builder form
(ii) roster form



What is its domain and range?

Ans. (i) Relation R in set-builder form is R = $\{(x, y): y = x - 2 : x = 5, 6, 7\}$

(ii) Relation R in roster form is R = {(5 3), (6, 4), (7, 5)}

Domain = {5, 6, 7}

Range = {3, 4, 5}

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by $\{(a, b): a, b \in A, b$ is exactly divisible by $a\}$.
(i) Write R in roster form.
(ii) Find the domain of R.
(iii) Find the range of R.

Ans. Given: A = {1, 2, 3, 4, 6}

A set of ordered pairs (a, b) where b is exactly divisible by a.

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (4, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

6. Determine the domain and range of the relation R defined by

R = $\{(x, x+5): x \in (0, 1, 2, 3, 4, 5)\}$

Ans. Given:

R= $\{(x, x+5): x \in (0, 1, 2, 3, 4, 5)\}$

= $\{(a, b): a = 0, 1, 2, 3, 4, 5\}$

$\therefore a = x$ and $b = x+5$

Putting $a = 0, 1, 2, 3, 4, 5$ we get $b = 5, 6, 7, 8, 9, 10$

$\therefore$ Domain of R = {0, 1, 2, 3, 4 5}

Range of R = {5,6,7,8,9,10}

7. Write the relation R = $\{(x, x^3): x$ **is a prime number less than** $10\}$ **in roster form.**

Ans. Given: R = $\{(x, x^3): x \text{ is a prime number less than } 10\}$

Putting x = 2, 3, 5, 7

R = {(2, 8), (3, 27), 5, 125), (7, 343)}

8. Let A = $\{x, y, z\}$ **and B = {1, 2}. Find the number of relations from A to B.**

Ans. Given: A = $\{x, y, z\}$ and B = {1, 2}

Number of elements in set A = 3 and Number of elements in set B = 2

$\therefore$ Number of subsets of $A \times B = 3 \times 2 = 6$

Number of relations from A to $B = 2^6$

9. Let R be the relation on Z defined by R = $\{(a, b): a, b \in Z^2, a - b$ **is an integer}. Find the domain and range of R.**

Ans. Given: R = $\{(a, b): a, b \in Z, a - b \text{ is an integer}\}$

= $\{(a, b): a, b \in Z, \text{ both } a \text{ and } b \text{ are even or both } a \text{ and } b \text{ are odd}\}$

= $\{(a,b): a,b \in Z, (a \text{ and } b \text{ are even}) \cup (a \text{ and } b \text{ are odd})\}$

$\therefore$ Domain of R = Z

Range of R = Z