Saket Shishu Ranjan Higher Secondary School, Vidisha
Class 11-Maths

Chapter 4- Trigonometry

Power Reducing Functions

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

$$cosec(x) = \frac{1}{sin(x)}$$

$$sec(x) = \frac{1}{cos(x)}$$

$$cot(x) = \frac{1}{tan(x)}$$

$$sin(x) = \frac{1}{cosec(x)}$$

$$cos(x) = \frac{1}{sec(x)}$$

$$tan(x) = \frac{1}{cot(x)}$$

Pythagorean Identities:

$$sin^2(x) + cos^2(x) = 1$$

$$cot^2 x + 1 = cosec^2 x$$

$$1 + tan^2 x = sec^2 x$$

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Trigonometric Ratio's of Common angles

Angles(A)	SinA	Cos A	TanA	Cosec A	Sec A	Cot A
0^0	0	1	0	Not defined	1	Not defined
30^0	1/2	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45^0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60^0	$\frac{\sqrt{3}}{2}$	1/2	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90^0	1	0	Not defined	1	Not defined	0

Trigonometry Formula for Complementary and supplementary angles

Angles(Θ)	Sin Θ	Cos Θ	Tan Θ	Cot Θ	Sec Θ	Cosec Θ
-x	-sin x	cos x	-tan x	-cot x	Sec x	-cosec x
90^0 -x	Cos x	sin x	Cot x	Tan x	Cosec x	Sec x
90^0 +x	Cos x	-sin x	-cot x	-tan x	-cosec x	Sec x
180 -x	Sin x	-cos x	-tan x	-cot x	-sec x	Cosec x
180 +x	-sin x	-cos x	Tan x	Cot x	-sec x	- cosec x
360 -x	- sin x	Cos x	-tan x	-cot x	Sec x	-cosec x
360 +x	sin x	Cos x	Tan x	Cot x	Sec x	Cosec x

Sin and cos function

1. $cos(A + B) = cos(A)cos(B) - sin(A)sin(B)$
2. $cos(A - B) = cos(A)cos(B) + sin(A)sin(B)$
3. $cos(\pi/2 - A) = sin(A)$
4. $sin(\pi/2 - A) = cos(A)$
5. $sin(A + B) = sin(A)cos(B) + sin(B)cos(A)$
6. $sin(A - B) = sin(A)cos(B) - sin(B)cos(A)$

Tan and cot functions

If none of the angles x, y and (x + y) is an odd multiple of $\pi/2$

$$tan(A + B) = \frac{tan(A)+tan(B)}{1-tan(A)tan(B)}$$

$$tan(A - B) = \frac{tan(A)-tan(B)}{1+tan(A)tan(B)}$$

If none of the angles x, y and (x + y) is an multiple of $\pi/2$

$$cot(A + B) = \frac{cot(A)cot(B)-1}{cot(A)+cot(B)}$$

$$cot(A - B) = \frac{cot(A)cot(B)+1}{cot(B)-cot(A)}$$

Some more Trigonometric Functions

Double of x (Double Of Angles)

$$cos2x = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x = \frac{1-tan^2x}{1+tan^2x}$$

$$sin2x = 2cos(x)sin(x) = \frac{2tan(x)}{1+tan^2x}$$

$$tan2x = \frac{2tan(x)}{1-tan^2x}$$

Triple of x (Triple of Angles)

$$sin3x = 3sin(x) - 4sin^3x$$

$$cos3x = 4cos^3x - 3cos(x)$$

$$tan(3x) = \frac{3tanx - tan^3x}{1-3tan^2x}$$

Sum and Difference of Angles

$$cos(A) + cos(B) = 2cos\frac{A+B}{2}cos\frac{A-B}{2}$$

$$cos(A) - cos(B) = -2sin\frac{A+B}{2}sin\frac{A-B}{2}$$

$$sin(A) + sin(B) = 2sin\frac{A+B}{2}cos\frac{A-B}{2}$$

$$sin(A) - sin(B) = 2cos\frac{A+B}{2}sin\frac{A-B}{2}$$

Half Angle Formula

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$$

$$\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$$

Pythagoras Identities in Radical form

$$\sin x = \pm\sqrt{1-\cos^2 x}$$

$$\tan x = \pm\sqrt{\sec^2 x - 1}$$

$$\cos x = \pm\sqrt{1-\sin^2 x}$$

Trigonometric equations Formula's

1.$sinx = 0$ implies $x = n\pi$, where n is any integer
2.$cosx = 0$ implies $x = (2n+1)(\pi/2)$

1. $sinx = siny$ then $x = n\pi + (-1)^n y$ where n is any integer
2. $cosx = cosy$ then $x = 2n\pi + y$ or $x = 2n\pi - y$ where n is any integer
3. $tanx = tany$ then $x = n\pi + y$ where n is any integer

5.Equation of the form
$sin^2 x = sin^2 y, cos^2 x = cos^2 y, tan^2 x =$
General solution is given by
$x = n\pi \pm y$ where n is any integer

6.Equation of the form
|sin x|=1 ,General solution is given by
$x = (2n+1)\frac{\pi}{2}$
|cos x|=1,General solution is given by
$x = n\pi$

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Practice Questions

Find the values of other trigonometric functions in exercises 1 to 5.

1. $\cos x = -\frac{1}{2}, x$ **lies in third quadrant.**

Ans. Given: $\cos x = -\frac{1}{2}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2 x + \left(-\frac{1}{2}\right)^2 = 1$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$

$\Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \sin x = -\frac{\sqrt{3}}{2}$ [x lies in third quadrant]

Now, $\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}}$

$\sec x = \frac{1}{\cos x} = -2$

$\tan x = \frac{\sin x}{\cos x} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

$\cot x = \frac{\cos x}{\sin x} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

2. $\sin x = \frac{3}{5}, x$ **lies in second quadrant.**

Ans. Given: $\sin x = \frac{3}{5}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{3}{5}\right)^2 + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$

$\Rightarrow \cos^2 x = \frac{16}{25}$

$\Rightarrow \cos x = \pm\frac{4}{5}$

$\Rightarrow \cos x = -\frac{4}{5}$ [x lies in second quadrant]

Now, $\operatorname{cosec} x = \frac{1}{\sin x} = \frac{5}{3}$

$\sec x = \frac{1}{\cos x} = -\frac{5}{4}$

$\tan x = \frac{\sin x}{\cos x} = \frac{3/5}{-4/5} = \frac{-3}{4}$

$\cot x = \frac{\cos x}{\sin x} = \frac{-4/5}{3/5} = \frac{-4}{3}$

3. $\cot x = \frac{3}{4}$, x **lies in third quadrant.**

Ans. Given: $\cot x = \frac{3}{4}$

$\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \operatorname{cosec}^2 x - \left(\frac{3}{4}\right)^2 = 1$

$\Rightarrow \operatorname{cosec}^2 x = 1 + \frac{9}{16}$

$\Rightarrow \operatorname{cosec}^2 x = \frac{25}{16}$

$\Rightarrow \operatorname{cosec} x = \pm\frac{5}{4}$

$\Rightarrow \operatorname{cosec} x = -\frac{5}{4}$ [x lies in third quadrant]

Now, $\sin x = \frac{1}{\operatorname{cosec} x} = \frac{-4}{5}$

$\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \frac{16}{25}} = -\frac{3}{5}$

$\tan x = \frac{1}{\cot x} = \frac{4}{3}$ $\sec x = \frac{1}{\cos x} = \frac{-5}{3}$

4. $\sec x = \frac{13}{5}, x$ **lies in fourth quadrant.**

Ans. Given: $\sec x = \frac{13}{5}$

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\frac{13}{5}\right)^2 - \tan^2 x = 1$

$\Rightarrow \tan^2 x = \left(\frac{13}{5}\right)^2 - 1$

$\Rightarrow \tan^2 x = \frac{169}{25} - 1$

$\Rightarrow \tan^2 x = \frac{144}{25}$

$\Rightarrow \tan x = \pm\frac{12}{5}$

$\Rightarrow \tan x = \frac{-12}{5}$ [x lies in fourth quadrant]

Now $\cot x = \frac{1}{\tan x} = \frac{-5}{12}$

$\cos x = \frac{1}{\sec x} = \frac{5}{13}$

$\sin x = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{144}{169}} = -\frac{12}{13}$

$\cos ec\ x = \frac{1}{\sin x} = \frac{-13}{12}$

5. $\tan x = \frac{-5}{12}, x$ **lies in second quadrant.**

Ans. Given: $\tan x = \frac{-5}{12}$

$\therefore \cot x = \frac{1}{\tan x} = \frac{-12}{5}$

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2 x - \left(\frac{-5}{12}\right)^2 = 1$

$\Rightarrow \sec^2 x = 1 + \frac{25}{144}$

$\Rightarrow \sec^2 x = \frac{169}{144}$

$\Rightarrow \sec x = \pm\frac{13}{12}$

$\Rightarrow \sec x = \frac{-13}{12}$ [x lies in second quadrant]

Now, $\cos x = \frac{1}{\sec x} = \frac{-12}{13}$

$$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{13}{5}$$

Find the values of the trigonometric functions in exercises 6 to 10.

6. $\sin 765^\circ$

Ans. Here $\sin 765^\circ = \sin\left(2\times 360^\circ + 45^\circ\right) = \sin 45^\circ = \frac{1}{\sqrt{2}}$

7. $\operatorname{cosec}(-1410)^\circ$

Ans. Here $\operatorname{cosec}(-1410)^\circ = \operatorname{cosec}\left(-4\times 360^\circ + 30^\circ\right) = \operatorname{cosec} 30^\circ = 2$

8. $\tan \frac{19\pi}{3}$

Ans. Here $\tan\frac{19\pi}{3} = \tan\frac{19}{3}\times 180^\circ = \tan 1140^\circ = \tan\left(3\times 360^\circ + 60^\circ\right) = \tan 60^\circ = \sqrt{3}$

9. $\sin\left(\frac{-11\pi}{3}\right)$

Ans. Here $\sin\left(\frac{-11\pi}{3}\right) = \sin\left(\frac{-11\times 180^\circ}{3}\right) = \sin\left(-660^\circ\right) = \sin\left(-2\times 360^\circ + 60^\circ\right) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

10. $\cot\left(\frac{-15\pi}{4}\right)$

Ans. Here $\cot\left(\frac{-15\pi}{4}\right) = \cot\left(\frac{-15\times 180^\circ}{4}\right) = \cot\left(-675^\circ\right) = \cot\left(-2\times 360^\circ + 45^\circ\right) = \cot 45^\circ = 1$

Prove that:

11. $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

Ans. 1. L.H.S. = $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

= $\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2=\frac{1}{4}+\frac{1}{4}-1$

= $\frac{1+1-4}{4}=\frac{-2}{4}=\frac{-1}{2}$ R.H.S.

12. $2\sin^2\frac{\pi}{6}+\cos ec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Ans. L.H.S. = $2\sin^2\frac{\pi}{6}+\cos ec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

= $2\sin^2\frac{\pi}{6}+\cos ec^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

= $2\sin^2\frac{\pi}{6}-\cos ec^2\frac{\pi}{6}\cos^2\frac{\pi}{3}$

= $2\times\left(\frac{1}{2}\right)^2+(-2)^2\times\left(\frac{1}{2}\right)^2$

= $2\times\frac{1}{4}+4\times\frac{1}{4}=\frac{6}{4}=\frac{3}{2}$

13. $\cot^2\frac{\pi}{6}+\cos ec\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Ans. L.H.S. = $\cot^2\frac{\pi}{6}+\cos ec\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

= $\cot^2\frac{\pi}{6}+\cos ec\left(\pi-\frac{\pi}{6}\right)+3\tan^2\frac{\pi}{6}$

= $\cot^2\frac{\pi}{6}+\cos ec\frac{\pi}{6}+3\tan^2\frac{\pi}{6}$

= $\left(\sqrt{3}\right)^2 2+3\times\left(\frac{1}{\sqrt{3}}\right)^2$

= $3+2+3\times\frac{1}{3}=5+1=6$ = R.H.S.

14. $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Ans. L.H.S. = $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

= $2\sin^2\left(\pi-\frac{\pi}{4}\right)+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

= $2\sin^2\frac{\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

= $2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times(2)^2$

= $2\times\frac{1}{2}+2\times\frac{1}{2}+2\times4=1+1+8=10$ = R.H.S.

15. Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Ans. (i) $\sin 75^\circ=\sin\left(45^\circ+30^\circ\right)=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$

= $\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15^\circ=\tan\left(45^\circ-30^\circ\right)=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}$

$$\left[\because \tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right]$$

$$=\frac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}}=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{3+1-2\sqrt{3}}{3-1}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$$

Prove the following:

16. $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Ans. L.H.S. = $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$$=\cos\left[\frac{\pi}{4}-x+\frac{\pi}{4}-y\right]$$

$$[\because \cos(x+y)=\cos x\cos y-\sin x\sin y]$$

$$=\cos\left[\frac{\pi}{2}-(x+y)\right]=\sin(x+y)$$ = R.H.S.

17. $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left[\frac{1+\tan x}{1-\tan x}\right]^2$

Ans. L.H.S. = $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

[Using $\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$, $\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$]

$= \dfrac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$ $= \dfrac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}}$

$= \dfrac{(1+\tan x)^2}{(1-\tan x)^2}$ = R.H.S.

18. $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Ans. L.H.S. = $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$

$= \dfrac{-\cos x.\cos x}{\sin x(-\sin x)}$ $= \dfrac{-\cos^2 x}{-\sin^2 x}=\cot^2 x$ = R.H.S.

19. $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$

Ans. L.H.S. = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$= \sin x.\cos x(\tan x+\cot x)$

$= \sin x.\cos x\left(\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}\right)$

$= \sin x.\cos x\left(\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)$

= 1 = R.H.S.

20. $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

Ans. L.H.S. = $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$

= $\cos[(n+1)x-(n+2)x]$

= $\cos[nx+x-nx-2x]$

= $\cos(-x)=\cos x$ = R.H.S.

21. $\cos\left(\frac{3\pi}{2}+x\right)-\cos\left(\frac{3\pi}{2}+x\right)=-\sqrt{2}\sin x$

Ans. L.H.S. = $\cos\left(\frac{3\pi}{2}+x\right)-\cos\left(\frac{3\pi}{2}+x\right)$

= $-2\sin\frac{3\pi}{4}\sin x$ = $-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$

= $-2\sin\frac{\pi}{4}\sin x$ = $-2\times\frac{1}{\sqrt{2}}\sin x$

= $-\sqrt{2}\sin x$ = R.H.S.

22. $\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x$

Ans. L.H.S. = $\sin^2 6x-\sin^2 4x$

= $\sin(6x+4x).\sin(6x-4x)$

$[\because \sin^2 x-\sin^2 y=\sin(x+y)\sin(x-y)]$

= $\sin 10x\sin 4x$ = R.H.S.

23. $\cos^2 2x-\cos^2 6x=\sin 4x\sin 8x$

Ans. L.H.S. = $\cos^2 2x-\cos^2 6x$

= $\sin(2x+6x).\sin(6x-2x)$

$[\because \cos^2 y - \cos^2 x = \sin(x+y)\sin(x-y)]$

= $\sin 8x \sin 4x$ = R.H.S.

24. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Ans. L.H.S. = $\sin 2x + 2\sin 4x + \sin 6x$

= $[\sin 4x + \sin 2x] + [\sin 6x + \sin 4x]$

= $2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + 2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)$

= $2\sin 2x \cos x + 2\sin 5x \cos x$

= $2\cos x[\sin 3x + \sin 5x]$

= $2\cos x\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

= $2\cos x[2\sin 4x.\cos x]$

= $4\cos^2 x \sin 4x$ = R.H.S.

25. $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

Ans. L.H.S. = $\cot 4x(\sin 5x + \sin 3x)$

= $\frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

= $\frac{\cos 4x}{\sin 4x}[2\sin 4x \cos x]$ = $2\cos 4x \cos x$

R.H.S. = $\cot 4x(\sin 5x - \sin 3x)$

$= \frac{\cos 4x}{\sin 4x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$

$= \frac{\cos 4x}{\sin 4x}[2\cos 4x \sin x]$ $= 2\cos 4x \cos x$

∴ L.H.S. = R.H.S

26. $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Ans. L.H.S. = $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+2x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$

$= \frac{-2\sin 7x \sin 2x}{2\cos 10x \sin 7x} = -\frac{\sin 2x}{\cos 10x}$ = R.H.S

27. $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Ans. L.H.S. = $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x+3x}{2}\right)}$

$= \frac{2\sin 4x}{2\cos 4x} = \tan 4x$ = R.H.S

28. $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\left(\frac{x-y}{2}\right)$

Ans. L.H.S. = $\dfrac{\sin x-\sin y}{\cos x+\cos y}$

= $\dfrac{2\cos\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)}{2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)}$

= $\dfrac{\sin\left(\dfrac{x-y}{2}\right)}{\cos\left(\dfrac{x-y}{2}\right)}=\tan\left(\dfrac{x-y}{2}\right)$ = R.H.S

29. $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Ans. L.H.S. = $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}$

= $\dfrac{2\sin\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}{2\cos\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}$

= $\dfrac{\sin 2x}{\cos 2x}=\tan 2x$ = R.H.S

30. $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Ans. L.H.S. = $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

= $\dfrac{-(\sin 3x-\sin x)}{-(\cos^2 x-\sin^2 x)}$

= $\dfrac{2\cos\left(\dfrac{3x+x}{2}\right)\sin\left(\dfrac{3x-x}{2}\right)}{\cos 2x}$

= $\frac{2\cos 2x \sin x}{\cos 2x} = 2\sin x$ = R.H.S

31. $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Ans. L.H.S. = $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

= $\frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$

= $\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$

= $\frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$

= $\frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$ = $\cot 3x$ = R.H.S

32. $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Ans. We know that $\cot 3x = \cot(2x + x)$

$\Rightarrow \cot 3x = \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}$

$\Rightarrow \cot 3x(\cot 2x + \cot x) = \cot 2x \cot x - 1$

$\Rightarrow \cot 3x \cot 2x + \cot 3x \cot x = \cot 2x \cot x - 1$

$\Rightarrow \cot 3x \cot 2x + \cot 3x \cot x - \cot 2x \cot x + 1 = 0$

$\Rightarrow \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

33. $\tan 4x = \dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Ans. L.H.S. = $\tan 4x = \dfrac{2\tan 2x}{1-\tan^2 2x}$

= $\dfrac{2.\dfrac{2\tan x}{1-\tan^2 x}}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$

= $\dfrac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}}$

= $\dfrac{4\tan x}{1-\tan^2 x}\times\dfrac{(1-\tan^2 x)^2}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$

= $\dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$ = R.H.S

34. $\cos 4x = 1-8\sin^2 x\cos^2 x$

Ans. L.H.S. = $\cos 4x = 1-2\sin^2 2x$

= $1-2(2\sin x\cos x)^2$

= $1-2(4\sin^2 x\cos^2 x)$

= $1-8\sin^2 x\cos^2 x$ = R.H.S

35. $\cos 6x = 32\cos^6 x-48\cos^4 x+18\cos^2 x-1$

Ans. L.H.S. = $\cos 6x = 2\cos^2 3x-1$

= $2\left[4\cos^3 x-3\cos x\right]^2-1$

= $2\left[16\cos^6 x+9\cos^2 x-24\cos^4 x\right]-1$

= $32\cos^6 x+18\cos^2 x-48\cos^4 x-1$

= $32\cos^6 x-48\cos^4 x+18\cos^2 x-1$ = R.H.S

36. Find the principal and general solutions of the following equations:

1. $\tan x=\sqrt{3}$

Ans. Given: $\tan x=\sqrt{3}$ Here x lies in first or third quadrant.

$\therefore \tan x=\tan 60^\circ$ or $\tan x=\tan\left(180^\circ+60^\circ\right)$

$\Rightarrow \tan x=\tan 60^\circ$ or $\tan x=\tan 240^\circ$

$\Rightarrow \tan x=\tan\frac{\pi}{3}$ or $\tan x=\tan\frac{4\pi}{3}$

Therefore, the principal solutions are $\frac{\pi}{3},\frac{4\pi}{3}$.

Now, $\tan x=\tan\frac{\pi}{3}$

$\Rightarrow x=n\pi+\frac{\pi}{3}$ where $n\in$ Z

37. $\sec x=2$

Ans. Given: $\sec x=2\Rightarrow \cos x=\frac{1}{2}$ Here x lies in first or fourth quadrant.

$\therefore \cos x=\cos 60^\circ$ or $\cos x=\cos\left(360^\circ-60^\circ\right)$

$\Rightarrow \cos x=\cos 60^\circ$ or $\cos x=\cos 300^\circ$

$\Rightarrow \cos x=\cos\frac{\pi}{3}$ or $\cos x=\cos\frac{5\pi}{3}$

Therefore, the principal solutions are $\frac{\pi}{3},\frac{5\pi}{3}$.

Now, $\cos x=\cos\frac{\pi}{3}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$ where $n \in$ Z

38. $\cot x = -\sqrt{3}$

Ans. Given: $\cot x = -\sqrt{3} \Rightarrow \tan x = \frac{-1}{\sqrt{3}}$ Here x lies in second or fourth quadrant.

$\therefore \tan x = -\tan 30^\circ = \tan(180^\circ - 30^\circ)$ or $\tan x = \tan(360^\circ - 60^\circ)$

$\Rightarrow \tan x = \tan 150^\circ$ or $\tan x = \tan 330^\circ$

$\Rightarrow \tan x = \tan \frac{5\pi}{6}$ or $\tan x = \tan \frac{11\pi}{6}$

Therefore, the principal solutions are $\frac{5\pi}{6}, \frac{11\pi}{6}$.

Now, $\tan x = -\tan \frac{\pi}{6}$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$ where $n \in$ Z

39. $\operatorname{cosec} x = -2$

Ans. Given: $\operatorname{cosec} x = -2 \Rightarrow \sin x = \frac{-1}{2}$ Here x lies in third or fourth quadrant.

$\therefore \sin x = -\sin 30^\circ = \sin(180^\circ + 30^\circ)$ or $\sin x = \sin(360^\circ - 30^\circ)$

$\Rightarrow \sin x = \sin 210^\circ$ or $\sin x = \sin 330^\circ$

$\Rightarrow \sin x = \sin \frac{7\pi}{6}$ or $\sin x = \sin \frac{11\pi}{6}$

Therefore, the principal solutions are $\frac{7\pi}{6}, \frac{11\pi}{6}$.

Now, $\sin x = -\sin \frac{\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \left(\frac{7\pi}{6}\right)$ where $n \in$ Z

40.. $\cos 4x = \cos 2x$

Ans. Given: $\cos 4x = \cos 2x$

$\Rightarrow 4x = 2n\pi \pm 2x, n \in$ Z

$\Rightarrow 4x - 2x = 2n\pi$ or $4x + 2x = 2n\pi, n \in$ Z

$\Rightarrow 2x = 2n\pi$ or $6x = 2n\pi, n \in$ Z

$\Rightarrow x = n\pi$ or $x = \frac{n\pi}{3}, n \in$ Z

Therefore, the principal solutions are $n\pi, \frac{n\pi}{3} n \in$ Z

41. $\cos 3x + \cos x - \cos 2x = 0$

Ans. Given: $\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0$ or $2\cos x - 1 = 0$

$\Rightarrow 2x = (2n+1)\frac{\pi}{2}$ or $\cos x = \frac{1}{2} = \cos\frac{\pi}{3}, n \in$ Z

$\Rightarrow x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}, n \in$ Z

42. $\sin 2x+\cos x=0$

Ans. Given: $\sin 2x+\cos x=0$

$\Rightarrow 2\sin x\cos x+\cos x=0$

$\Rightarrow \cos x(2\sin x+1)=0$

$\Rightarrow \cos x=0$ or $2\sin x+1=0$

$\Rightarrow x=(2n+1)\frac{\pi}{2}$ or $\sin x=\frac{-1}{2}=-\sin\frac{\pi}{6}, n\in$ Z

$\Rightarrow x=(2n+1)\frac{\pi}{2}$ or $x=n\pi+(-1)^n\left(\frac{-\pi}{6}\right), n\in$ Z

$\Rightarrow x=(2n+1)\frac{\pi}{2}$ or $x=n\pi+(-1)^n\frac{7\pi}{6}, n\in$ Z

43. $\sec^2 2x=1-\tan 2x$

Ans. Given: $\sec^2 2x=1-\tan 2x$

$\Rightarrow 1+\tan^2 2x=1-\tan 2x$

$\Rightarrow \tan 2x(\tan 2x+1)=0$

$\Rightarrow \tan 2x=0$ or $\tan 2x+1=0$

$\Rightarrow 2x=n\pi$ or $\tan 2x=-1=-\tan\frac{\pi}{4}$

$\Rightarrow x=\frac{n\pi}{2}$ or $x=\frac{n\pi}{2}+\frac{3\pi}{8}, n\in$ Z

44. $\sin x+\sin 3x+\sin 5x=0$

Ans. Given: $\sin x+\sin 3x+\sin 5x=0$

$\Rightarrow 2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)+\sin 3x=0$

$\Rightarrow 2\sin 3x\cos 2x+\sin 3x=0$

$\Rightarrow \sin 3x(2\cos 2x+1)=0$

$\Rightarrow \sin 3x=0$ or $2\cos 2x+1=0$

$\Rightarrow 3x=n\pi$ or $\cos 2x=\frac{-1}{2}=\cos\frac{2\pi}{3}, n\in$ Z

$\Rightarrow x=\frac{n\pi}{3}$ or $2x=2n\pi\pm\frac{2\pi}{3}, n\in$ Z

$\Rightarrow x=\frac{n\pi}{3}$ or $x=n\pi\pm\frac{\pi}{3}, n\in$ Z

Prove that:

45.

$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Ans. L.H.S. = $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

= $\cos\left(\frac{9\pi}{13}+\frac{\pi}{13}\right)+\cos\left(\frac{9\pi}{13}+\frac{\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

= $\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

= $\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

= $-\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$ = 0 = R.H.S

46. $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x=0$

Ans. L.H.S. = $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x$

= $\left[2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]\sin x+\left[-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right]\cos x$

= $[2\sin 2x\cos x]\sin x+[-2\sin x\sin x]\cos x$

= $2\sin 2x\cos x\sin x-2\sin x\sin x\cos x$ = 0 = R.H.S.

47. $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Ans. L.H.S. = $(\cos x+\cos y)^2+(\sin x-\sin y)^2$

= $\left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]^2+\left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2$

= $4\cos^2\left(\frac{x+y}{2}\right)\cos^2\left(\frac{x-y}{2}\right)+4\cos^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)$

= $4\cos^2\left(\frac{x+y}{2}\right)\left[\cos^2\left(\frac{x-y}{2}\right)+\sin^2\left(\frac{x-y}{2}\right)\right]$

= $4\cos^2\left(\frac{x+y}{2}\right)$ = R.H.S.

48. $(\cos x-\cos y)^2+(\sin x-\sin y)^2=4\sin^2\frac{x-y}{2}$

Ans. L.H.S. = $(\cos x-\cos y)^2+(\sin x-\sin y)^2$

= $\left[-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2+\left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2$

= $4\sin^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)+4\cos^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)$

= $4\sin^2\left(\frac{x-y}{2}\right)\left[\sin^2\left(\frac{x+y}{2}\right)+\cos^2\left(\frac{x+y}{2}\right)\right]$

= $4\sin^2\left(\frac{x-y}{2}\right)$ = R.H.S.

49. $\sin x+\sin 3x+\sin 5x+\sin 7x=4\cos x\cos 2x\sin 4x$

Ans. L.H.S. = $\sin x+\sin 3x+\sin 5x+\sin 7x$ $(\sin 7x+\sin x)+(\sin 5x+\sin 3x)$

= $\left[2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right)\right]+\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

= $[2\sin 4x\cos 3x]+[2\sin 4x\cos x]$

= $2\sin 4x[\cos 3x+\cos x]$

= $2\sin 4x\left[2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]$

= $2\sin 4x[2\cos 2x\cos x]$

= $4\cos x\cos 2x\sin 4x$ = R.H.S.

50. $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$

Ans. L.H.S. = $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

= $\frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)\right]}$

= $\dfrac{2\sin 6x\cos x + 2\sin 6x\cos 3x}{2\cos 6x\cos x + 2\cos 6x\cos 3x}$

= $\dfrac{2\sin 6x(\cos x + \cos 3x)}{2\cos 6x(\cos x + \cos 3x)}$

= $\tan 6x$ = R.H.S.

51. $\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Ans. L.H.S. = $\sin 3x + \sin 2x - \sin x$ = $(\sin 3x - \sin x) + \sin 2x$

= $\left[2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right] + 2\sin x\cos x$

= $2\cos 2x\sin x + 2\sin x\cos x$

= $2\sin x[\cos 2x + \cos x]$

= $2\sin x\left[2\cos\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right)\right]$

= $2\sin x\left[2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)\right]$

= $4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$ = R.H.S.